A NUMERICAL INVESTIGATION OF COMBINED SHOCK AND VIBRATION ISOLATION THROUGH THE SEMI-ACTIVE CONTROL OF MAGNETORHEOLOGICAL FLUID DAMPER IN PARALLEL WITH AN AIR SPRING

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ABSTRACT

Combining shock and vibration isolation into a single isolation mount is investigated numerically through the use of the Bouc-Wen model of a magnetorheological fluid damper in parallel with an air spring. The stability and dissipative capabilities of the Bouc-Wen model are proven mathematically. The response characteristics of this hybrid isolator to shock and vibration inputs is explored. The advantages of combining shock and vibration isolation into a single package is discussed.

I: INTRODUCTION

A classic problem in isolation system design is the design of a single isolator that can perform equally well as a shock and a vibration isolator. This problem arises due to the fact that a good vibration isolator tends to be a poor shock isolator and a good vibration isolator tends to be a poor shock isolator. Most attempts to solve the combined isolation problem with a passive device quickly lead to frustration. This is certainly the case in shipboard isolation applications where many inputs are often present simultaneously. The typical approach to solving the shipboard isolation problem involves separate passive isolators for shock and vibration. This inevitably leads to modifying vibration isolators to survive shock inputs and/or modifying shock isolators to perform adequately as vibration isolators.

When many different inputs are present, as in shipboard applications, an active, or semi-active, isolation system is justified. Active, or semi-active, isolation systems can be designed to simultaneously isolate for many combined and varying inputs. The advantage to combining shock and vibration isolation into a single package is obvious. A single combined isolation system replaces two separate systems and therefore reduces weight and increases available volume. This is particularly important in submarine applications where space is limited. Also, with varying and diverse inputs, a combined active isolation system can be designed to perform better than separate passive shock and vibration isolation systems operating in parallel.

For shipboard applications the ideal isolation system would provide effective vibration isolation in the 10 to 200 Hz range and shock isolation from a variety of inputs such as underwater explosions, wave slap, impact, etc. Thus a "soft" isolator with a natural frequency of approximately 2Hz is desired that can survive varying shock inputs. An ideal vibration isolator that meets this design criteria is an air spring. Air springs have a low natural frequency, but

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tend to behave poorly as shock isolators due to their inherently low damping. An ideal combined isolator would be an air spring that could provide high, variable damping forces on demand during a shock event. Producing high, variable damping forces is an ideal application for a magnetorheological (MR) fluid damper. Therefore, combining an air spring with an MR damper in parallel would provide an ideal combined isolation system provided the optimal damping force can be applied and varied rapidly enough to attenuate the shock input and meet rattle space requirements.

To investigate the effectiveness of an air spring/MR damper combination as a combined isolation system requires an accurate model of the highly nonlinear MR damper. One such model that very accurately predicts the response of the MR damper is the so-called Bouc-Wen model. The stability and dissipative capabilities of the Bouc-Wen model can be proven mathematically as will be shown. Since this paper is a numerical study, the MR damper under consideration will be a commercially available model developed by the Lord Corporation (Model RD-1003) which has been adequately characterized in several papers [2-3]. The air spring under consideration is also a commercially available model manufactured by Firestone (Model 20). The air spring was chosen as it's physical dimensions and capacity are a good match for the MR damper. The force-deflection behavior of the air spring was provided by Firestone and incorporated into the model.

II: SYSTEM MODEL

The system under consideration will be modeled as a single-degree-of-freedom (SDOF) system consisting of a modified Bouc-Wen model with an additional spring and damper to model the air spring. The complete model is shown in Figure (1).

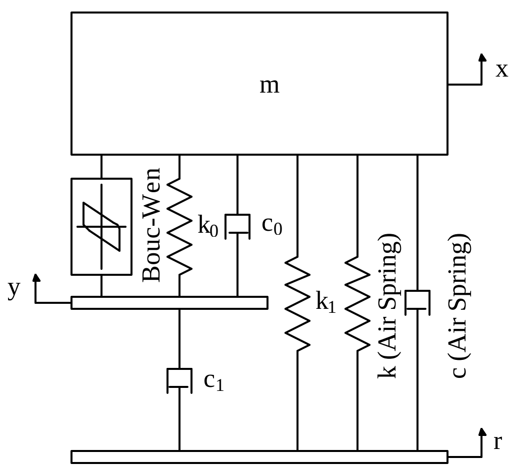

Figure 1: SDOF System Model

The core of the model is the modified Bouc-Wen model to which several papers have been devoted [2-3]. The next section discusses the stability and dissipative capabilities of the modified Bouc-Wen model.

III: STABILITY OF THE MODIFIED BOUC-WEN MODEL

The stability of the modified Bouc-Wen model will be addressed by using the SDOF model shown in Figure (1). In the following discussion the air spring stiffness and damping shown in the figure are not included. The dynamics of the system can be described as follows:

$$m\ddot{x} = \alpha z + k_0\left(y - x\right) + c_0\left(\dot{y} - \dot{x}\right) + k_1\left(r - x\right) \quad (1a)$$

$$c_1\left(\dot{r} - \dot{y}\right) = \alpha z + k_0\left(y - x\right) + c_0\left(\dot{y} - \dot{x}\right) \quad (1b)$$

$$\dot{z} = -\gamma\left|\dot{y} - \dot{x}\right| z \left|z\right|^{n-1} - \beta\left(\dot{y} - \dot{x}\right)\left|z\right|^{n} + A\left(\dot{y} - \dot{x}\right) \quad (1c)$$

Putting into state space form by letting $x_1 = x$, $x_3 = y$, and $x_4 = z$, we get

$$\dot{x}_1 = x_2 \tag{2a}$$

$$\dot{x}_2 = -\frac{k_0}{m}(x_1 - x_3) - \frac{k_1}{m}x_1 - \frac{c_0}{m}(x_2 - g) + \frac{\boldsymbol{a}}{m}x_4 - \ddot{r} \tag{2b}$$

$$\dot{x}_3 = \frac{1}{c_0 + c_1}\left[k_0(x_1 - x_3) + c_0 x_2 - \boldsymbol{a}x_4\right] = g \tag{2c}$$

$$\dot{x}_4 = -g\left|g - x_2\right|\left|x_4\right|\left|x_4\right|^{n-1} - b(g - x_2)\left|x_4\right|^n + A(g - x_2) \tag{2c}$$

where,

$$g - x_2 = \frac{1}{c_0 + c_1}\left[k_0 x_1 - c_1 x_2 - k_0 x_3 - \boldsymbol{a}x_4\right] \tag{3}$$

Finding the equilibrium point $x = 0$, yields:

$$0 = x_2 \tag{4a}$$

$$0 = -\frac{k_0}{m}(x_1 - x_3) - \frac{k_1}{m}x_1 + \frac{\boldsymbol{a}}{m}x_4 \tag{4b}$$

$$0 = 0 \tag{4c}$$

$$0 = k_0(x_1 - x_3) - \boldsymbol{a}x_4 \tag{4d}$$

$$\therefore x_1 = 0 \tag{4e}$$

$$0 = -k_0 x_3 - \boldsymbol{a}x_4 \tag{4f}$$

$$\therefore x_3 = -\frac{\boldsymbol{a}}{k_0}x_4 \tag{4g}$$

Realizing that the first three states are linear in their makeup, the model can be thought of as being composed of two subsystems, one linear in the first three states, and one nonlinear in the fourth. Substitution and manipulation puts the states into the following form:

$$\dot{x}_1 = x_2 \tag{5a}$$

$$\dot{x}_2 = \frac{1}{m}\left[-(k_0 + k_1) + \frac{k_0 c_0}{c_0 + c_1}\right]x_1 - \frac{c_0 c_1}{m(c_0 + c_1)}x_2 + \frac{1}{m}\left(k_0 - \frac{k_0 c_0}{c_0 + c_1}\right)x_3 + \frac{1}{m}\left(1 - \frac{c_0}{c_0 + c_1}\right)x_4 - \ddot{r} \tag{5b}$$

$$\dot{x}_3 = \frac{k_0 c_0}{c_0 + c_1}x_1 + \frac{c_0}{c_0 + c_1}x_2 - \frac{k_0}{c_0 + c_1}x_3 - \frac{\boldsymbol{a}}{c_0 + c_1}x_4 = g \tag{5c}$$

$$\dot{x}_4 = -g\left|g - x_2\right|\left|x_4\right|\left|x_4\right|^{n-1} - b(g - x_2)\left|x_4\right|^n + A(g - x_2) \tag{5d}$$

This result leads the stability search into the passivity formalism [1] for linear systems. We can now look at the whole system as a composite of two subsystems, one linear and the other nonlinear, that transfer energy between one another. Putting the equations into feedback form, the input to the linear system, u_1, will be the negative of the output of the nonlinear system, x_4, (i.e. $y_2 = x_4$, $u_1 = -y_2$) [1], while the output of the linear system will be $y_1 = g - x_2$, which will be the input to the nonlinear system ($u_2 = y_1$). This results in the following system of equations:

$$\underline{\dot{x}}_1 = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{1}{m}\left[k_1 + k_0\left(1-\frac{c_0}{c_0+c_1}\right)\right] & -\frac{1}{m}\left(\frac{c_0 c_1}{c_0+c_1}\right) & \frac{1}{m}\left[k_0\left(1-\frac{c_0}{c_0+c_1}\right)\right] \\ \frac{k_0}{c_0+c_1} & \frac{c_0}{c_0+c_1} & -\frac{k_0}{c_0+c_1} \end{bmatrix}\underline{x}_1 + \begin{bmatrix} 0 \\ \frac{a}{m}\left(-1+\frac{c_0}{c_0+c_1}\right) \\ \frac{a}{c_0+c_1} \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}\ddot{r} \tag{6a}$$

$$y_1 = \begin{bmatrix} \frac{k_0}{c_0+c_1} & -\frac{c_1}{c_0+c_1} & -\frac{k_0}{c_0+c_1} \end{bmatrix}\underline{x}_1 + \begin{bmatrix} \frac{a}{c_0+c_1} \end{bmatrix} u_1 \tag{6b}$$

where,

$$x = [x_1 \quad x_2 \quad x_3]^T \tag{7a}$$
$$u_1 = -x_4 \tag{7b}$$
$$y_1 = g - x_2 \tag{7c}$$
$$\dot{x}_4 = -g|u_2|x_4|x_4|^{n-1} - bu_2|x_4|^n + Au_2 \tag{7d}$$
$$u_2 = y_1 \tag{7e}$$
$$y_2 = u_1 \tag{7f}$$

The passivity theory formalism states that Lyapunov functions are additive, where Lyapunov functions of subsystems can be combined to describe an overall system Lyapunov function [1]. This is derived from the fact that Lyapunov functions are generally composed of terms representing the energy of the system.

The prior form of the derivative of a Lyapunov function, which will be of use in this proof of stability, is based on the conservation of energy equation of the form:

[Stored Energy] = [External Power Input] + [Internal Power Generation] (8)

The external power input term can be represented as $y^T u$ of an input u and an output y. In this way of thinking, the derivative of the Lyapunov function of a subsystem, can have the following form:

$$\dot{V}_i = \mathbf{y}_i^T \mathbf{u}_i - g_i(t) \tag{9}$$

where $\dot{V}_i$ and $g_i(t)$ are scalar functions of time, u_i is the subsystem input, and $y_i(t)$ the output. In this case, there are two subsystems, and therefore two Lyapunov functions to be determined (V_1 and V_2). The goal is to get both of their respective derivatives in power form in order to prove that the system is not only passive, but dissipative as well.

For the linear subsystem 1 (states x_1, x_2, and x_3), $y_1 = g - x_2$, $u_1 = -x_4 = y_2$, the Lyapunov function is represented by:

$$V_1 = \frac{1}{2}mx^2 + \frac{1}{2}k_0(y-x)^2 + \frac{1}{2}k_1(r-x)^2 \tag{10}$$

which upon substitution and dropping the input term r becomes:

$$V_1 = \frac{1}{2}mx_2^2 + \frac{1}{2}k_0(x_1 - x_3)^2 + \frac{1}{2}k_1 x_1^2 \tag{11}$$

in state space. Taking the derivative of V_1 leads to, after substitution and manipulation

$$\dot{V}_1 = -\frac{c_0 c_1}{c_0 + c_1} x_2^2 - \frac{k_0^2 (x_1 - x_3)^2}{c_0 + c_1} + \left[\frac{a k_0}{c_0 + c_1} (x_1 - x_3) + a\left(-1 + \frac{c_0}{c_0 + c_1} \right) x_2 \right] u_1 \tag{12}$$

To put $\dot{V} = y_1^T u_1 - g_1(t)$ form, recall

$$y_1 = \left[\frac{k_0}{c_0 + c_1} x_1 - \frac{c_1}{c_0 + c_1} x_2 - \frac{k_0}{c_0 + c_1} x_3 + \frac{a}{c_0 + c_1} u_1 \right] \tag{13}$$

Adding $a\left(y_1^T u_1 - y_1^T u_1\right)$ to the above result for $\dot{V}$ results in

$$\begin{aligned} \dot{V}_1 = & -\frac{c_0 c_1}{c_0 + c_1} x_2^2 - \frac{k_0^2 (x_1 - x_3)^2}{c_0 + c_1} + \left[\frac{a k_0}{c_0 + c_1} (x_1 - x_3) + a\left(-1 + \frac{c_0}{c_0 + c_1} \right) x_2 \right] u_1 \\ & - \frac{a k_0}{c_0 + c_1} x_1 u_1 + \frac{a c_1}{c_0 + c_1} x_2 u_1 + \frac{a k_0}{c_0 + c_1} x_3 u_1 - \frac{a^2}{c_0 + c_1} u_1^2 \end{aligned} \tag{14}$$

which after manipulation, results in

$$\dot{V}_1 = a y_1^T u_1 - \frac{c_0 c_1}{c_0 + c_1} x_2^2 - \frac{1}{c_0 + c_1} \left[k_0 (x_1 - x_3) + a u_1 \right]^2 \tag{15}$$

Letting

$$g_1(t) = \frac{c_0 c_1}{c_0 + c_1} x_2^2 + \frac{1}{c_0 + c_1} \left[k_0 (x_1 - x_3) + a u_1 \right]^2 \tag{16}$$

we get the power form, therefore

$$\dot{V}_1 = a y_1^T u_1 - g_1(t) \tag{17}$$

where $g_1(t) \geq 0$ for all $t \geq 0$. Therefore, V_1 is not only passive, but dissipative since [1]

a) $\int y_1^T u_1 \, dt \neq 0$ (18a)

b) $\int g_1(t) > 0$ (18b)

The passivity of the subsystem is preserved if multiplied by a strictly positive number since this simply scales the first term. Turning our attention to the nonlinear subsystem, a Lyapunov function is represented by:

$$V_2 = \frac{1}{2} x_4^2 \tag{19}$$

Taking the derivative of V_2 with respect to time results in

$$\dot{V}_2 = Px_4 u_2 - g\left|u_2\right|x_4^2\left|x_4\right|^{n-1} \tag{20}$$

where $P = A - b\left|x_4\right|^n$. With $y_2 = x_4$,

$$\dot{V}_2 = Py_2 u_2 - g_2(t) \tag{21}$$

where,

$$g_2(t) = g\left|u_2\right|x_4^2\left|x_4\right|^{n-1}; \quad g_2(t) \geq 0 \tag{22}$$

Now the question is if $P > 0$. It has been proven that due to the mathematics involved in the Bouc-Wen model, $\left|x_4\right|$ saturates at a maximum value

$$\left|x_4\right| \leq \left[\frac{A}{g+b}\right]^{\frac{1}{n}} \tag{23}$$

At worst case,

$$\left|x_4\right| = \left[\frac{A}{g+b}\right]^{\frac{1}{n}} \tag{24a}$$

$$\therefore P = A - b\left\{\left[\frac{A}{g+b}\right]^{\frac{1}{n}}\right\}^n = A - b\left[\frac{A}{g+b}\right] = A\left[1 - \frac{b}{g+b}\right] \geq 0 \tag{24b}$$

Therefore, $\dot{V}_2 = Py_2 u_2 - g_2(t)$ is passive.

Since,

a) $\int y_2^T u_2 dt \neq 0$ (25a)

b) $\int g_2(t) dt > 0$ (25b)

V_2 is dissipative.

Thus,

$V_T = V_1 + V_2$ is positive definite (26a)

$\dot{V}_T = \dot{V}_1 + \dot{V}_2$ is dissipative since both subsystems are dissipative. (27b)

Simulations of a nominal type system (all parameters set to one) shows the system's dissipative ability when subjected to an impulsive input as shown in Figure (2) as well as different sinusoidal inputs with different amplitudes and frequencies as shown in Figures (3).

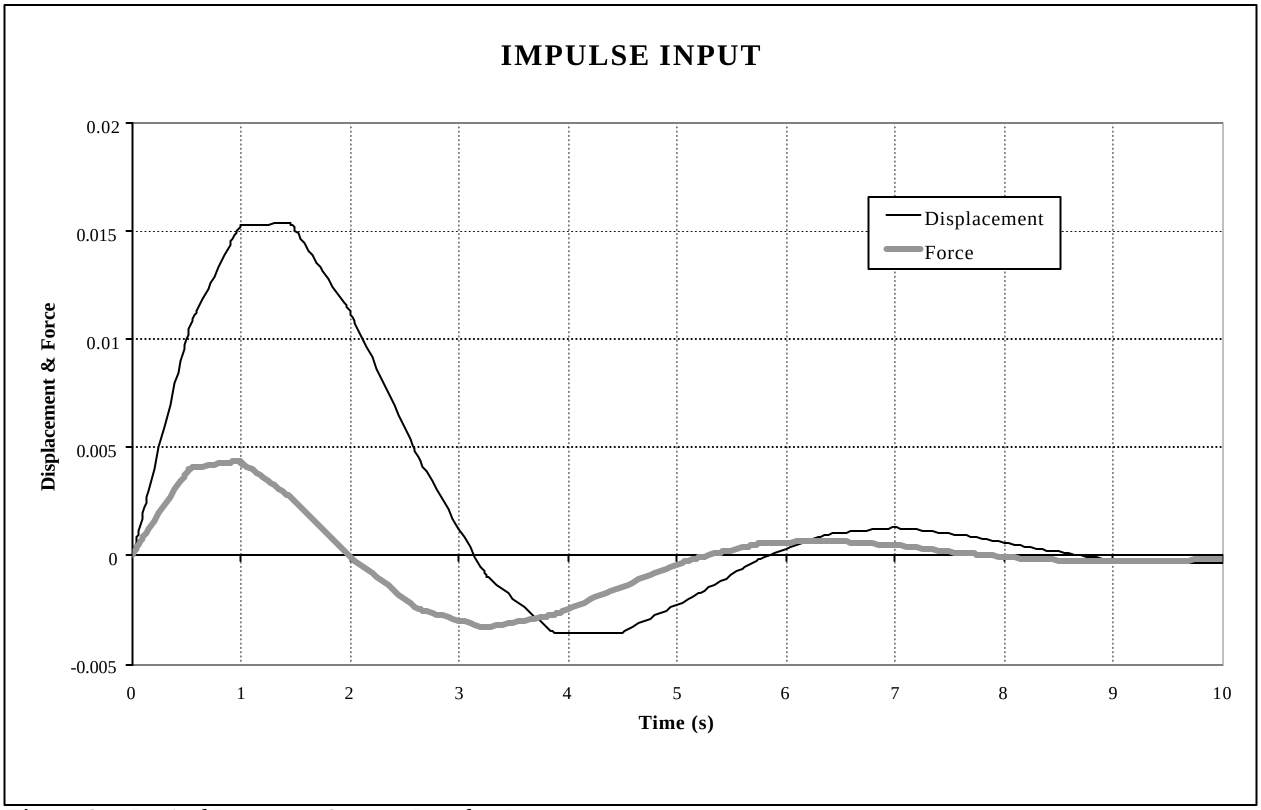

Figure 2: Nominal Parameter System: Impulse Response

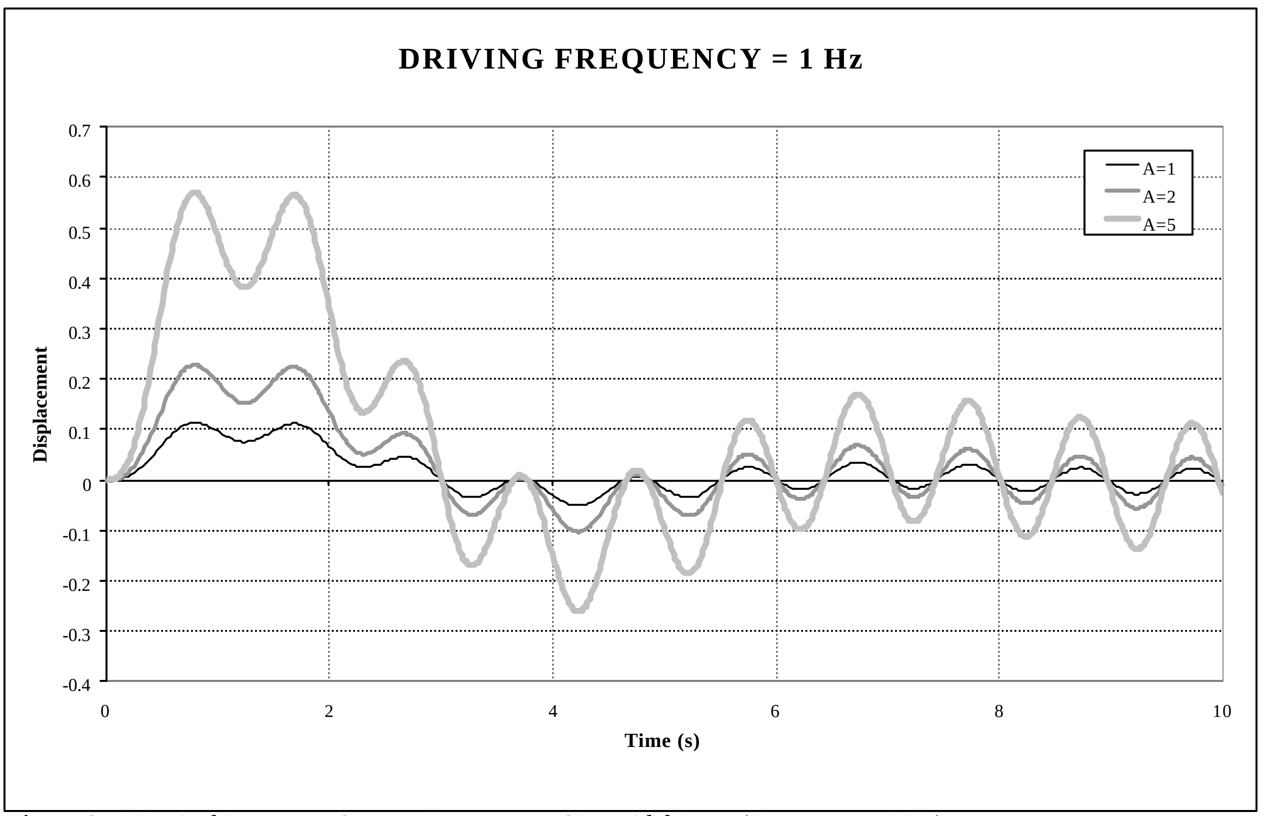

Figure 3a: Nominal Parameter System: Response to Sinusoidal Input (Frequency = 1 Hz)

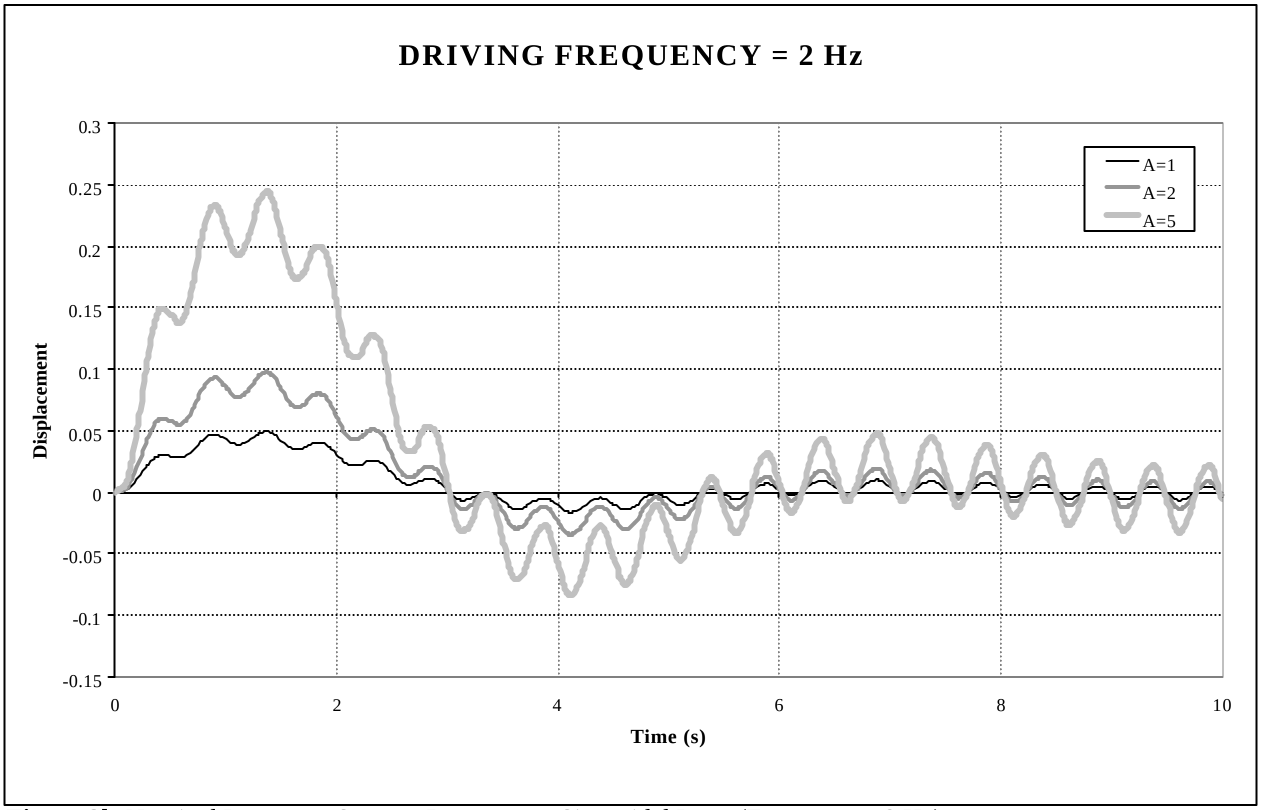

Figure 3b: Nominal Parameter System: Response to Sinusoidal Input (Frequency = 2 Hz)

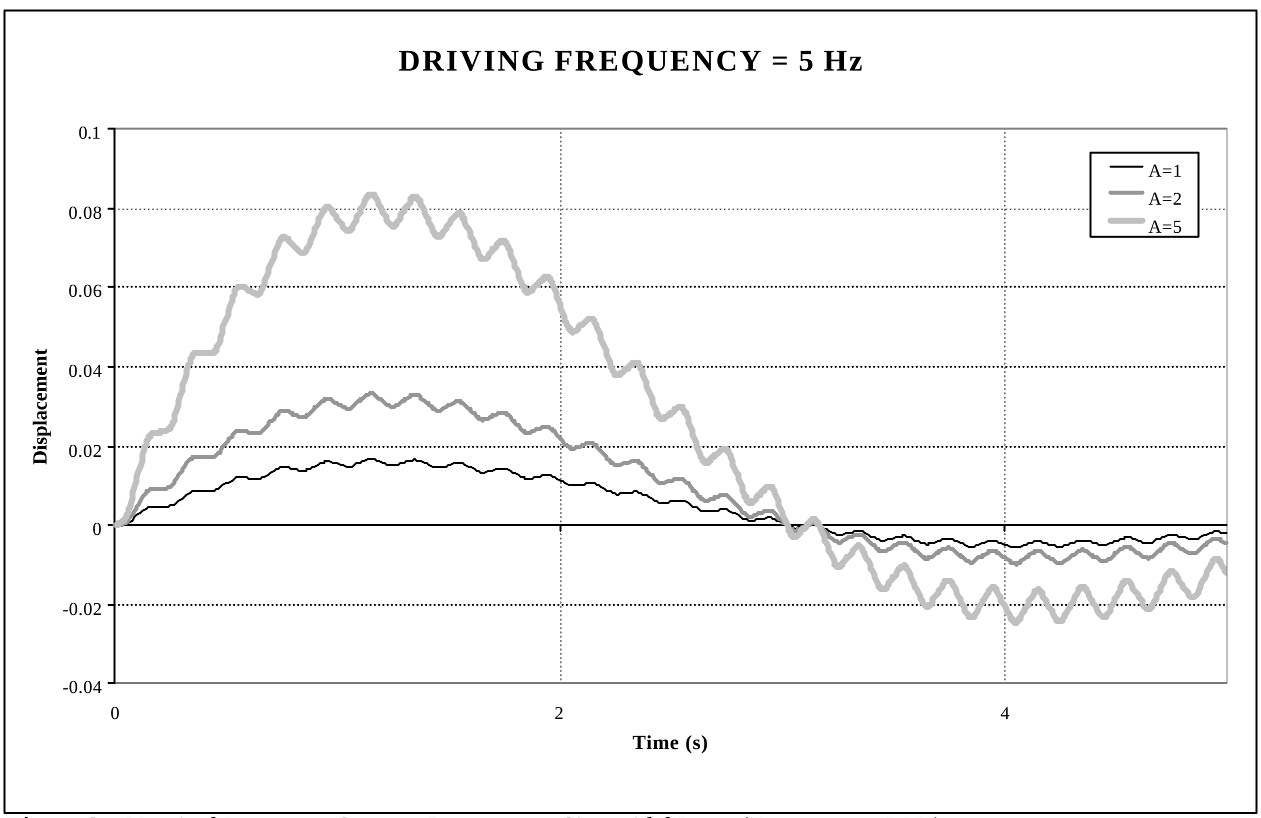

Figure 3c: Nominal Parameter System: Response to Sinusoidal Input (Frequency = 5 Hz)

IV: CONTROL ALGORITHMS

Many control algorithms have been suggested in the literature [2-3] for control of semi-active devices. Here four control algorithms were considered that have been successfully implemented for control of the MR damper: Control based on Lyapunov stability theory, Decentralized Bang-Bang control, Modulated Homogeneous Friction and Clipped-Optimal control. These controllers have been discussed in numerous papers [2-3] on semi-active control and therefore in the interest of brevity will not be discussed again here. All of the following numerical simulations were performed using MATLAB® and/or Simulink®[2].

V: VIBRATION ISOLATION PERFORMANCE

In order to examine the vibration isolation performance of the combined isolator, the force transmissibility was plotted for the air spring only and for the air spring with the addition of the MR damper in passive off (i.e., zero volts applied to the current driver) mode. This was done to show the magnitude of the negative effect of adding the MR damper on the vibration isolation effectiveness. No voltage was applied to the damper since the most effective vibration isolation performance will be achieved with minimum damping. The input to the model was an above mount sinusoidal force of varying driving frequencies. The transmissibility ratio was then plotted versus the frequency ratio as shown in Figure (4).

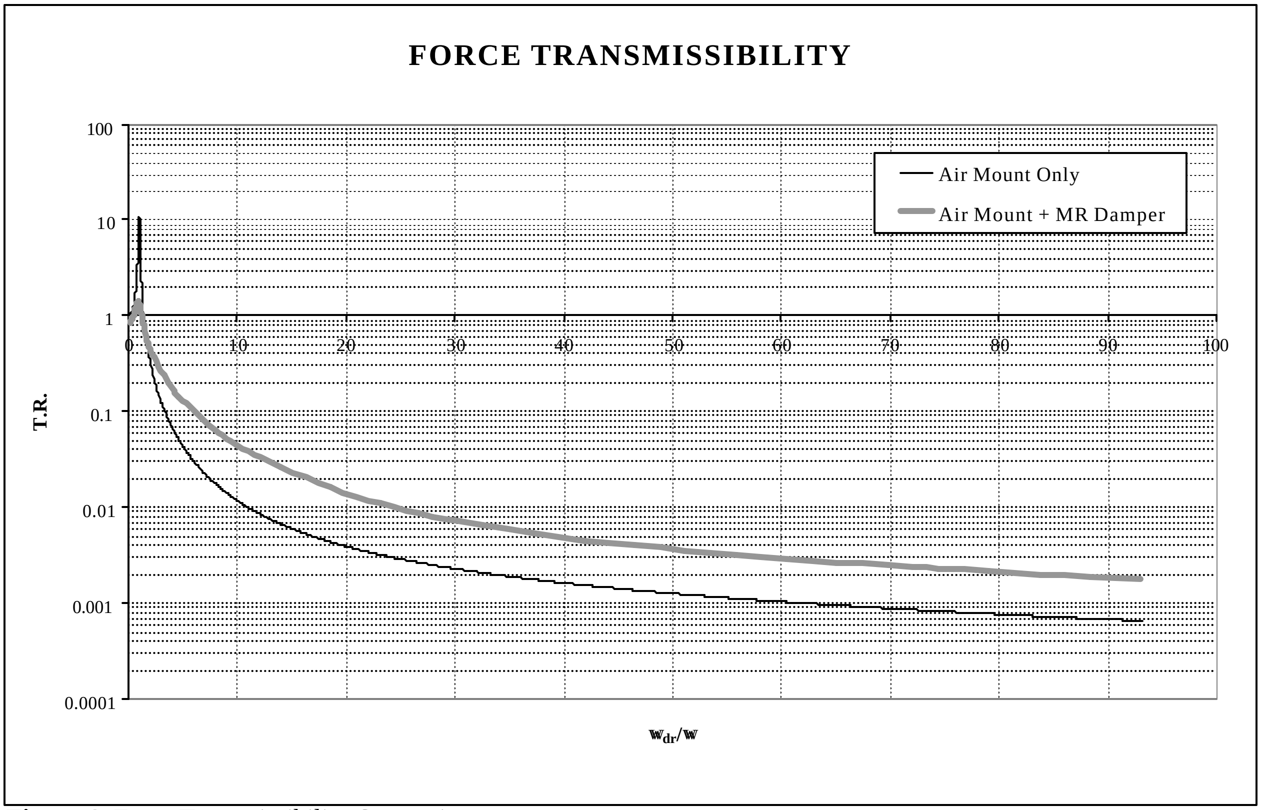

Figure 4: Force Transmissibility Comparison

From Figure (4) it is clear that the addition of the MR damper does not seriously degrade the vibration isolation performance when compared to the air mount without the damper. This result is surprising as the commercial MR damper is fairly stiff and uses high friction seals. Note that even with the MR damper, the combined isolator is very effective over the 10 - 200 Hz vibration isolation range of interest which corresponds to a frequency ratio of approximately 4.65 - 93.0 based on the 2.15 Hz natural frequency of the air mount.

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[2] MATLAB® AND Simulink® are registered trademarks of The Mathworks, Inc.

VI: SHOCK ISOLATION PERFORMANCE

In an effort to determine the effectiveness of the air spring/MR damper combination as a shock isolator, the model was subjected to a base excitation which was derived from a known deck response (Floating Shock Platform with 13 Hz Deck Fixture) due to an underwater explosion. The input was scaled to a 1g peak to avoid exceeding the peak force output of the damper. The response displacement, velocity and acceleration was recorded for the air mount without the MR damper and for each of the controllers. Figure (5) shows the base input used in the tests.

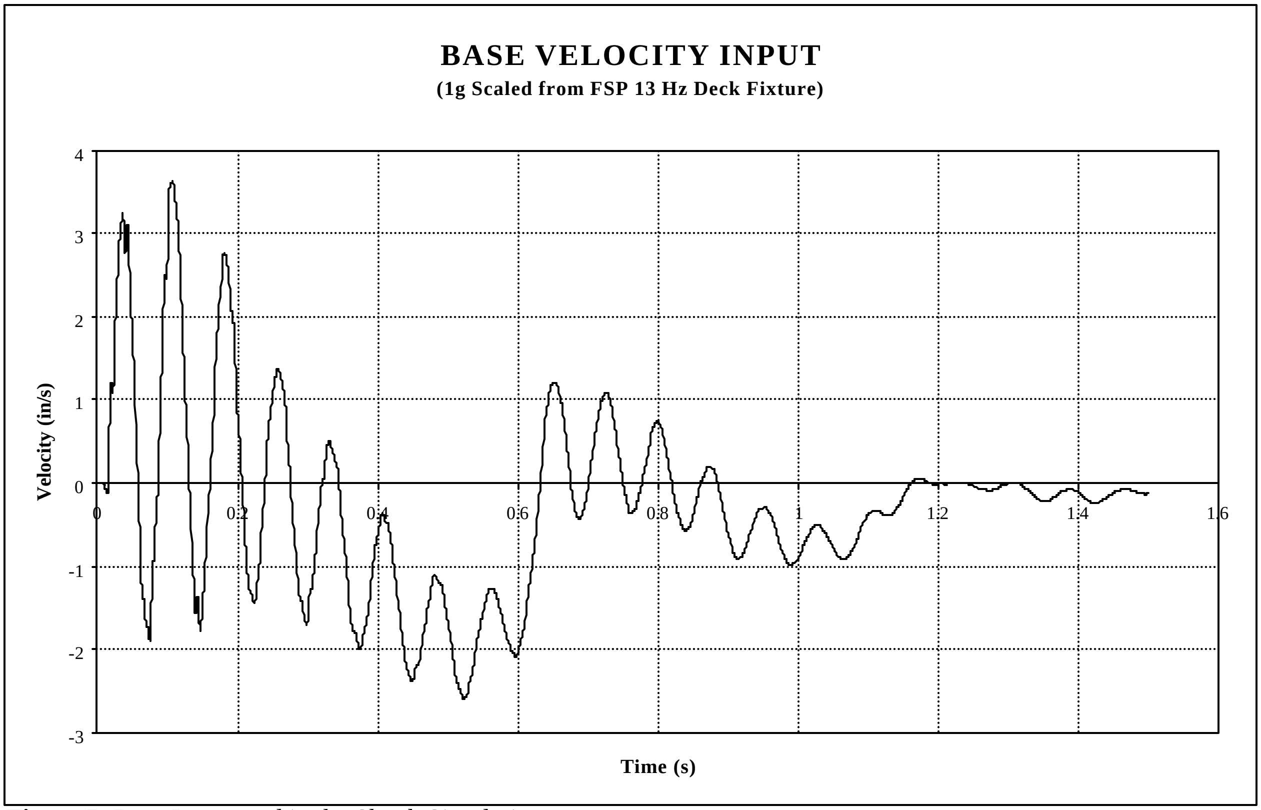

Figure 5: Base Input used in the Shock Simulations

Figure (6) shows the relative displacement across the mount and Figure (7) shows the above mount accelerations in g's. Figure (8) summarizes the peak response values.

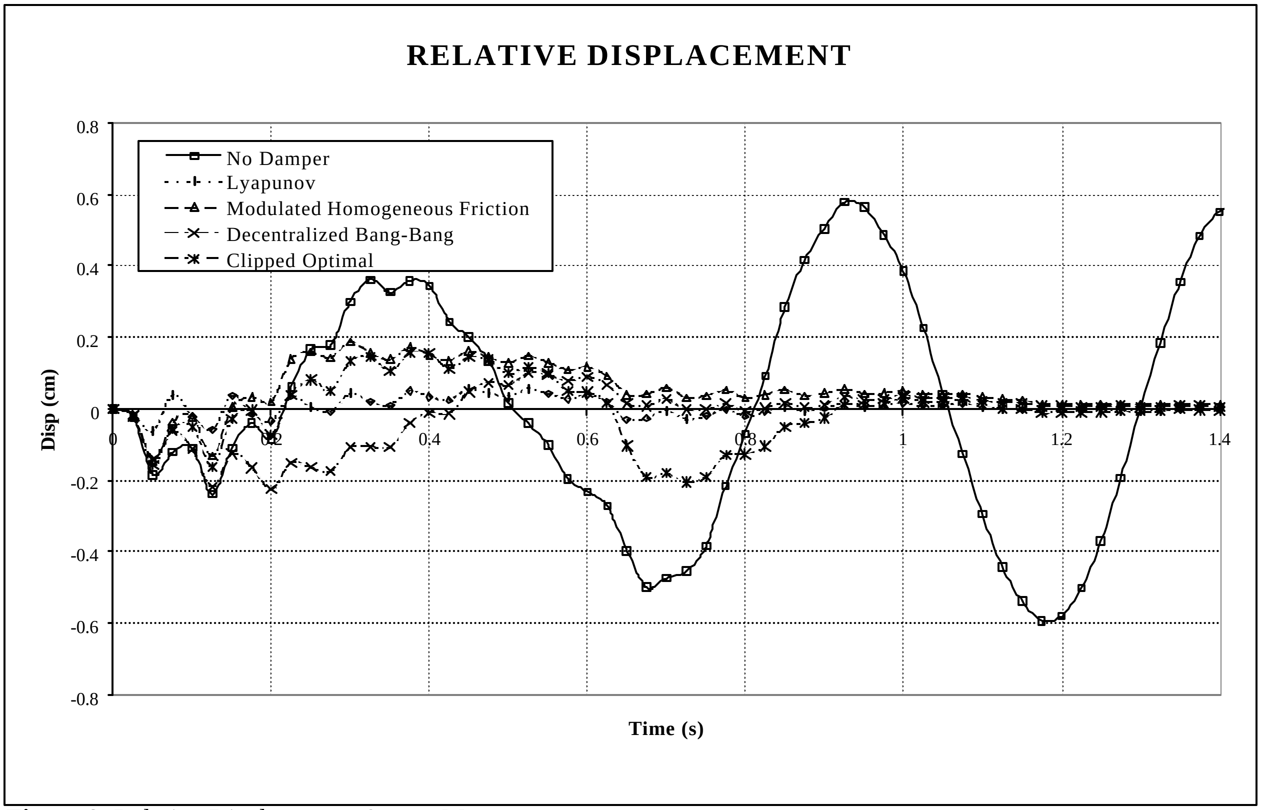

Figure 6: Relative Displacement Across Mount

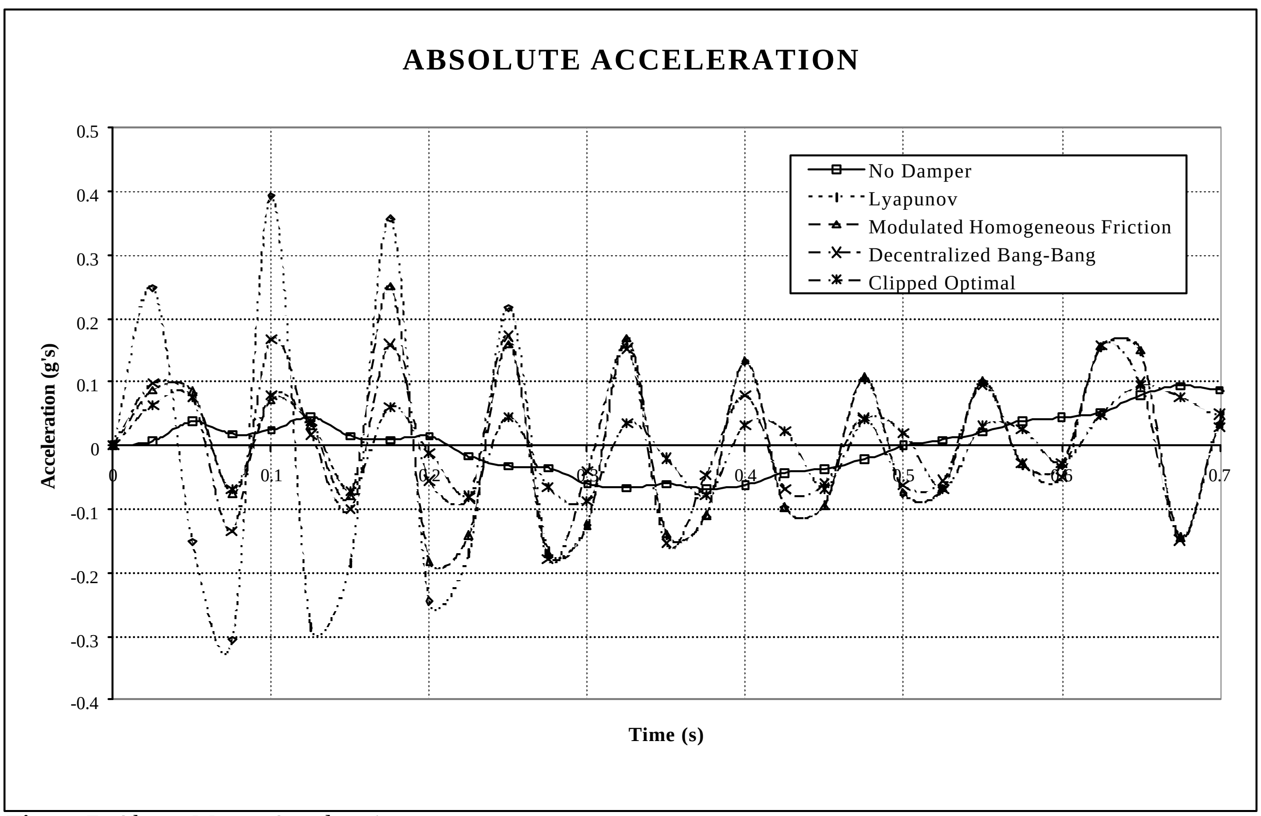

Figure 7: Above Mount Acceleration

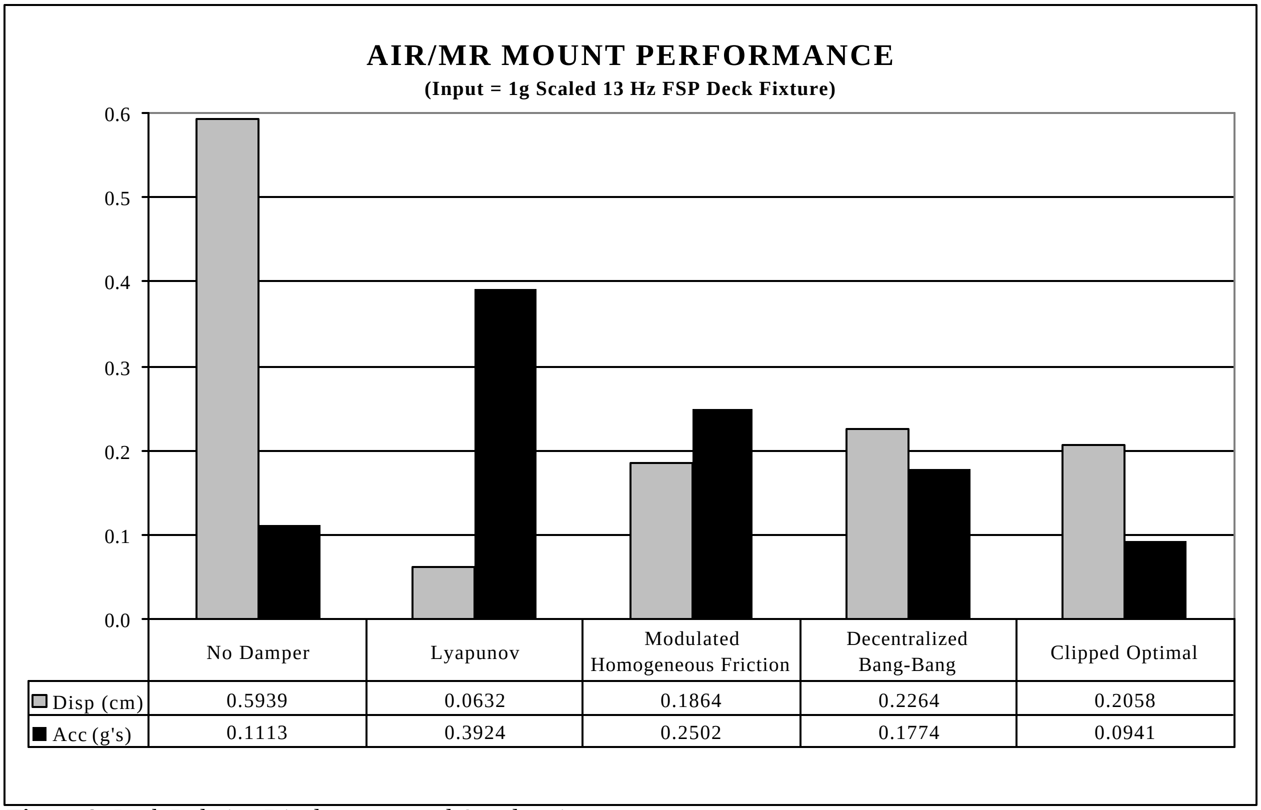

Figure 8: Peak Relative Displacement and Acceleration

By looking at Figure (8) it is clear that the controllers offer a significant improvement in terms of reducing relative displacement across the mount. This can be critical if a particular application has a tight rattle space requirement. This is often the case in shipboard applications, particularly submarine applications, where space is at a premium. Although three of the controllers increase above mount acceleration over the air spring without a damper they still offer a significant reduction in transmitted g's. Table (1) shows the acceleration transmissibility, or percent transmitted g's, for each controller.

CONTROL ALGORITHM	ACCELERATION TRANSMISSIBILITY
Lyapunov	0.3924
Modulated Homogeneous Friction	0.2502
Decentralized Bang-Bang	0.1774
Clipped Optimal	0.0941

Table 1: Summary of Acceleration Transmissibility for the Controllers

Particularly effective is the Clipped-Optimal controller which offers a significant reduction in above mount g's and a relatively small peak displacement across the mount. Note that the air mount without the damper offers a significant reduction in above mount g's, but it does so at the expense of a large relative displacement. In many applications this excessive displacement may be unacceptable.

VII: SUMMARY AND CONCLUSION

The modified Bouc-Wen model of the MR damper provides a very accurate model for use in numerical simulations.
As was shown, the stability and dissipative capabilities of the modified Bouc-Wen model can be proven mathematically. The stability of the modified Ben-Wouc model was proven using the passivity formalism, in which the mathematical description could be broken down into two subsystems, one linear and one nonlinear. This passivity approach was valuable in the stability analysis since the other techniques results in inconclusive results.

Some of these techniques researched were the direct Lyapunov approach, Kalman-Yacubovitch theorem, linear passivity proof, among others.

Putting both the subsystems into feedback form, in which the output of one subsystem is used as the input to the other, and vice versa, the time derivative of the respective Lyapunov function could be placed in the power form. The power form representation, which can be written as an external power term plus an internal energy dissipation term, was used to prove that the two subsystems were both passive and dissipative (based on the internal energy dissipation term being negative for two subsystems).

Since the two subsystems Lyapunov functions are additive, their derivatives are as well. Since both subsystems are passive as well as dissipative, the summation of the two composing the entire system shows it as passive and dissipative as well. Computer simulation of an impulsive and persisting sinusoidal input on a nominal model (all system parameters set to one) show stability. The impulse shows exponential decaying type response indicative of dissipative systems. The sine input results show the expected response of dissipative systems, where the output profiles reach a steady state response as time increases.

An air mount can be a very effective vibration isolator, but tends to be a poor shock isolator due to its inherently low damping. Air mounts can offer a significant reduction in transmitted acceleration, but tend to do so through large relative displacements. In many applications these large mount deflections are unacceptable. Adding a controlled MR damper offers a significant reduction in mount deflection (Up to 89.4%) and a significant reduction in transmitted acceleration (Up to 90.6%). Preliminary work with multi-degree-of-freedom models show even more significant reductions in mount deflection and transmitted g's.

Combining vibration and shock isolation into a single unit can offer significant advantages in terms of weight and space savings. This is particularly important in submarine applications where space is at a premium. Although this study looked at a scaled mount, the vibration and shock performance characteristics of this hybrid mount apply in a relative sense to higher capacity mounts. Thus the subject mount could conceivable be scaled for use as a high capacity shock mount in shipboard applications.

Although semi-active devices require a power source, MR devices are attractive in that they have low power requirements and are inherently stable, i.e., without power they still behave as passive dampers. Another benefit of having low power requirements is that it may be possible to create a self-powered device. Having a self-powered, self-contained, combined shock and vibration isolator would be especially attractive as it would eliminate the need for external power sources and external controller connections. Future work will look at just this possibility.

VIII: ACKNOWLEDGEMENTS

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IV: REFERENCES

[1] Slotine, J.J. and Li, W., 1991, *Applied Nonlinear Control*, Prentice Hall, Inc., Englewood Cliffs, New Jersey

[2] Spencer Jr., B.F., Dyke, S.J., Sain, M.K. and Carlson, J.D., "Phenomenological Model of a Magnetorheological Damper", *Proceedings of the IASTED International Conference, Intelligent Information Systems*, The Bahamas, (1997).

[3] Dyke, S.J. and Spencer Jr., B.F., "A Comparison of Semi-Active Control Strategies for the MR Damper", *ACSE Journal of Engineering Mechanics*, (1996)